# A PROCEDURE FOR THE DESIGN OF MICROWAVE FILTERS BASED ON A DISTRIBUTED PARAMETER MODEL

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**Abstract**–A synthesis procedure based on a distributed parameter model for both narrow-band and broad-band microwave filters is presented. The frequency response of the filter is described in terms of the characteristic polynomial $T_{21} = S_{11}/S_{21}$ where $S_{11}$ and $S_{21}$ are the scattering parameters of the filter. Starting from the desired polynomial $T_{21}$, the design scheme directly yields the scattering parameters of the various junctions and the length of the resonators. On the basis of this technique, a Chebyshev-type 8 pole E-plane filter has been designed and built. The excellent agreement between the predicted and the measured data confirm the validity of this synthesis procedure.

## 1 Introduction

In the past [1]-[9] and in recent years [10]-[16] a great deal of effort has been devoted to the design of microwave filters. Most of the synthesis techniques that have been developed are based on a low-pass prototype, from which the corresponding microwave filter is obtained. Often, in the case of broad-band filters, a numerical optimization procedure is then necessary to satisfy the filter specifications.

The synthesis scheme presented in this paper, is based on a distributed parameter model of the filter. As is well known, an $N$-resonator microwave filter can be seen as a cascade of $N+1$ junctions interconnected by $N$ transmission lines corresponding to the fundamental waveguide mode. The $N+1$ junctions (i.e. irises, thick slots, E-plane septa, etc.) are conveniently described in terms of their scattering matrices. Starting from this distributed model, this synthesis procedure directly yields the scattering parameters of the various junctions, which can be obtained by any kind of discontinuity. From this point of view coaxial-waveguide transitions are included in the definition of the input/output junctions. The procedure is based on the properties of the characteristic polynomial, which is the element $T_{21} = S_{11}/S_{21}$ of the transmission matrix, with $S_{11}$ and $S_{21}$ being the scattering parameters of the filter. An arbitrary frequency response of the filter can be obtained by suitably positioning the $T_{21}$ roots, which correspond to the reflection zeros of the filter.

The synthesized frequency response fails to completely match the specifications for some broad-band filters. This is mainly due to the frequency dispersion of the $S$-parameters, the multimodal interactions between the various junctions and the losses of the material, all neglected in the synthesis procedure. However, the degradations introduced by all these causes can be approximately described, at least in the frequency range of interest, as produced by a suitable linear system, whose characteristics are obtained by a system indentification technique. This allows to modify the design goal in such a way that the synthesized response meets perfectly the specifications.

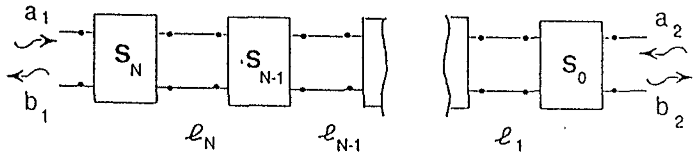


Figure 1: Two-port network equivalent circuit of a N-resonator filter.

## 2 Synthesis procedure

As is shown in Fig.1, a $N$-pole microwave filter can be described by an equivalent two port network, where $\mathbf{S}^{(k)}$, $(k = 0 \sim N)$ is the scattering matrix of the $k$-th junction and $l_k$, $(k = 1 \sim N)$ the length of the transmission line corresponding to the $k$-th resonator. Consider the transmission matrix of the $k$-th discontinuity. In the case of a reciprocal and lossless structure, this matrix can be written as follows:

$$\mathbf{T}^{(k)} = \begin{bmatrix} 1/S_{21}^{(k)} & -S_{22}^{(k)}/S_{21}^{(k)} \\ S_{11}^{(k)}/S_{21}^{(k)} & -\Delta S^{(k)}/S_{21}^{(k)} \end{bmatrix} = e^{-j\phi_{21}^{(k)}} \begin{bmatrix} 1 & 0 \\ 0 & e^{j\phi_{11}^{(k)}} \end{bmatrix}$$

$$\cdot \begin{bmatrix} \csc\gamma_k & \cot\gamma_k \\ \cot\gamma_k & \csc\gamma_k \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -e^{j\phi_{22}^{(k)}} \end{bmatrix} \quad (1)$$

where $\cos\gamma_k = |S_{11}^{(k)}|$, $\sin\gamma_k = |S_{21}^{(k)}|$ and $\phi_{ij}^{(k)}$ are the phases of the scattering parameter $S_{ij}^{(k)}$ and $\Delta S_{ij}^{(k)}$ is the determinant of $\mathbf{S}^{(k)}$. If $\beta$ is the propagation constant of the waveguide of the length $l_k$ which constitutes the $k$-th cavity, the corresponding transmission matrix can be written as:

$$\mathbf{T}^{(k)} = e^{j\beta l_k} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2j\beta l_k} \end{bmatrix} \quad (2)$$

According to the expressions given in (1) and (2), the transmission matrix of the whole filter can be expressed (apart from phase terms) as follows:

$$\mathrm{T} = \prod_{k=N,1} \left\{ \begin{bmatrix} \csc\gamma_k & \cot\gamma_k \ z^{-1} e^{-j\psi_k} \\ \cot\gamma_k & \csc\gamma_k \ z^{-1} e^{-j\psi_k} \end{bmatrix} \right\} \begin{bmatrix} \csc\gamma_0 & \cot\gamma_0 \\ \cot\gamma_0 & \csc\gamma_0 \end{bmatrix} , \quad (3)$$

where one has introduced the complex variable $z$, defined as:

$$z \ \exp\{j\psi_k\} = -\exp\{j[2\beta l_k - (\phi_{11}^{(k-1)} + \phi_{22}^{(k)})]\} \quad . \quad (4)$$

The unknown phase term $\psi_k$ has been introduced in order to account for the different phase behaviour of the resonators and to maintain a certain degree of generality in the synthesis procedure. Note that $z$ is not a function of the index $k$ and its phase is related to the frequency through the propagation constant $\beta$ and the phases of the scattering parameters of the junctions.

From (3) it is easy to recognize that the elements of the transmission matrix $\mathbf{T}$ are $N$-degree polynomials in the complex variable $z^{-1}$. In particular:

$$T_{11}(z) = \sum_{k=0}^{N} a_k \ z^{-k} \ ; \quad T_{21}(z) = \sum_{k=0}^{N} b_k \ z^{-k} \quad . \quad (5)$$

The element $T_{21} = S_{11}/S_{21}$ is the characteristic polynomial of the filter, and efficiently describes the frequency response both in the pass-band (where $T_{21} \sim S_{11}$) and in the stop-band (where $T_{21} \sim 1/S_{21}$). The polynomial $T_{21}$ can be interpreted as the array factor of a linear distribution of radiators or, alternatively, as the response of a digital filter. According to the first interpretation, the reflection coefficient of the filter in the pass-band corresponds to the level of the secondary lobes of the array factor, whereas the maximum insertion loss in the stop-band corresponds to the main lobe level. According to the second interpretation, the polynomial $T_{21}$ can be identified as the $Z$-transform of the impulse response of a FIR digital filter. In both cases well established synthesis techniques can be used to obtain the desired array factor [17] or the FIR transfer function [18]. Once the polynomial $T_{21}$ has been defined, according to the required specifications, an extraction procedure is applied to determine the scattering matrix of the various junctions.

In order to carry out the extraction procedure it is also necessary to know the element $T_{11} = 1/S_{21}$ of the transmission matrix of the filter.

For this purpose one recalls that, in the case of a reciprocal and lossless structure, the difference between the squared magnitude of $T_{11}$ and $T_{21}$ is one for $|z| = 1$, i.e. for real values of frequency:

$$|T_{11}(z)|^2 = 1 + |T_{21}|^2 \quad \forall |z| = 1 \ . \quad (6)$$

This relationship can be analitically continued in the whole complex plane $z$ by noting that $z^* = z^{-1}$ on the circle $|z| = 1$. Hence, the following relationship holds for any $z$:

$$T_{11}(z)\, T_{11}^*(1/z^*) = 1 + T_{21}(z)\, T_{21}^*(1/z^*) \quad \forall z. \quad (7)$$

On the basis of this equation, $T_{11}(z)$ can be determined from a specified $T_{21}(z)$. It is in fact easy to recognize that the $2N$ roots of the first member of (7) occur in pairs $\{\alpha_k, 1/\alpha_k^*\}$, where $\alpha_k$ are the $N$ roots of the polynomial $T_{11}$. The identification of the $\alpha_k$ is very simple noting that they are the poles of $S_{21}$ and hence lie inside the circle $|z| = 1$ because of the stability condition. The magnitude of the $N$-th degree coefficient ($a_N$) can be determined by evaluating (7), for example in $z = -1$, while its phase, as shown in what follows, must be equal to that of the coefficient of the polynomial $T_{21}$ with the same degree ($b_N$).

Once the polynomials $T_{11}^{[N]}$ e $T_{21}^{[N]}$ are determined (the superscript $[N]$ has been added to emphasize that they refer to the whole structure consisting of $N$ cavities), it is possible from (3) to write the following relationship:

$$\begin{bmatrix} T_{11}^{[N]} \\ T_{21}^{[N]} \end{bmatrix} = \begin{bmatrix} \csc\gamma_N & \cot\gamma_N \ z^{-1} \ e^{-j\psi_N} \\ \cot\gamma_N & \csc\gamma_N \ z^{-1} \ e^{-j\psi_N} \end{bmatrix} \begin{bmatrix} T_{11}^{[N-1]} \\ T_{21}^{[N-1]} \end{bmatrix} , \quad (8)$$

where the polynomials $T_{11}^{[N-1]}$ and $T_{21}^{[N-1]}$ correspond to the structure consisting of the first $N-1$ cavities. By solving the linear system (8)

$$\begin{bmatrix} T_{11}^{[N-1]} \\ T_{21}^{[N-1]} \end{bmatrix} = \begin{bmatrix} \csc\gamma_N & -\cot\gamma_N \\ -\cot\gamma_N \ z \ e^{-j\psi_N} & \csc\gamma_N \ z \ e^{-j\psi_N} \end{bmatrix} \begin{bmatrix} T_{11}^{[N]} \\ T_{21}^{[N]} \end{bmatrix} \quad (9)$$

the following relationships can be obtained from among the coefficients of the four polynomials involved:

$$\begin{aligned} a_k^{[N-1]} &= \csc\gamma_N \ a_k^{[N]} - \cot\gamma_N \ b_k^{[N]} \\ b_k^{[N-1]} &= [-\cot\gamma_N \ a_{k+1}^{[N]} + \csc\gamma_N \ b_{k+1}^{[N]}] \ e^{j\psi_N} \quad . \end{aligned} \quad (10)$$

Since the polynomials $T_{11}^{[N-1]}$ and $T_{21}^{[N-1]}$ are of degree $N-1$ (in the variable $z^{-1}$), the coefficients $a_N^{[N-1]}$ and $b_{-1}^{[N-1]}$ must be zero. By enforcing these conditions, two relationships are obtained, both giving the magnitude of $S_{11}$ of the $N$-th discontinuity:

$$|S_{11}^{[N]}| = \cos\gamma_N = \frac{a_N^{[N]}}{b_N^{[N]}} = \frac{b_0^{[N]}}{a_0^{[N]}} \quad . \quad (11)$$

Since this quantity must be real and positive, the phase of $a_N^{[N]}$ must be chosen equal to that of $b_N^{[N]}$, as previously stated. Under this condition, it can be shown from (7), that the second ratio always concides with the first.

The procedure described can be iterated to obtain the $S_{11}$ parameter of the $(N-1)$-th discontinuity as follows:

$$|S_{11}^{[N-1]}| = \cos\gamma_{N-1} = \frac{b_0^{[N-1]}}{a_0^{[N-1]}} = \frac{-\cos\gamma_N \ a_1^{[N]} + b_1^{[N]}}{a_0^{[N]} - \cos\gamma_N \ b_0^{[N]}} \ e^{j\psi_N} \quad . \quad (12)$$

From (12) the role played by the phase term $\psi_N$ introduced in (4) clearly appears. Its value must in fact be chosen so that the ratio (12) is real and positive.

On the basis of the previous considerations, the following iterative extraction scheme of the quantities $\gamma_k \in [0, \pi/2]$,

$(k = 0 \sim N)$ and $\psi_k$, $(k = 1 \sim N)$, can be identified:

$$p = N \sim 1$$

$$|S_{11}^{[p]}| = \cos\gamma_p = \frac{b_0^{[p]}}{a_0^{[p]}}$$

$$\psi_p = \mathcal{ARG}\left\{\frac{a_0^{[p]} - \cos\gamma_p\, b_0^{[p]}}{-\cos\gamma_p\, a_1^{[p]} + b_1^{[p]}}\right\}$$

$$k = 0 \sim p-1$$

$$a_k^{[p-1]} = \csc\gamma_p\, a_k^{[p]} - \cot\gamma_p\, b_k^{[p]}$$

$$b_k^{[p-1]} = [-\cot\gamma_p\, a_{k+1}^{[p]} + \csc\gamma_p\, b_{k+1}^{[p]}]\, e^{j\psi_p}$$

$$p = 0$$

$$|S_{11}^{[0]}| = \cos\gamma_0 = \frac{b_0^{[0]}}{a_0^{[0]}}$$

As in all extraction procedures (see for example [19]), particular attention must be paid to the numerical implementation of the algorithm. If the number of cavities increases and the bandwidth decreases so that the range of the polynomial $T_{11}$ on the circle $|z| = 1$ becomes large, round off errors can greatly affect the extraction procedure, expecially in the case of the last cells. In these cases the use of multiple precision can be necessary.

As far as the computation of the length of the cavities and the correspondence between electrical and angular bandwidth is concerned, it is necessary to refer to the expression of the phase of the complex variable $z$ defined in (4):

$$\theta = \pi + 2\beta(\omega)\, l_k - 2\phi_k(\omega) - \psi_k \qquad (13)$$

where, for brevity, the following variable has been introduced:

$$\phi_k(\omega) = \frac{1}{2}\,[\phi_{11}^{(k-1)}(\omega) + \phi_{22}^{(k)}(\omega)] \qquad (14)$$

If $\omega_1$ and $\omega_2$ are the angular frequencies of the two limits of the pass-band which correspond to the angles $\theta_1 = \pi - \Delta\theta_B/2$ and $\theta_2 = \pi + \Delta\theta_B/2$ respectively, from (13) one obtains:

$$l_k = \frac{\psi_k + \phi_k(\omega_1) + \phi_k(\omega_2)}{\beta(\omega_1) + \beta(\omega_2)} \qquad (15)$$

and for the angular bandwidth:

$$\Delta\theta_B = 2[\beta(\omega_2) - \beta(\omega_1)]l - 2[\phi(\omega_2) - [\phi(\omega_1)] \qquad (16)$$

where $l$ and $\phi$ are the mean values of the quantities $l_k$ and $\phi_k$, respectively. From these two expressions it is clear that, for the definiton of the angular bandwidth, necessary to identify the polynomial $T_{21}$ that must be synthesized, one has to estimate $\phi(\omega_1)$ and $\phi(\omega_2)$.

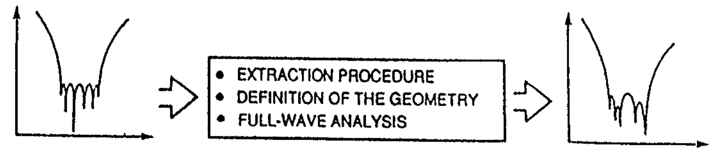


Figure 2: Linear system interpretation of the synthesis process.

In some cases, degradation effects, such as the frequency dispersion of the $S$-parameters, the multimodal interaction between the junctions and the losses are significant so that the frequency response obtained by the full-wave analysis does not completely match the specifications. These degradations can be described in terms of a linear system as shown in Fig.2. The key point of the method is to determine the fitting polynomial of the full-wave analysis so that, as least in the band of interest, the frequency response given by the full-wave analysis, can be described as follows:

$$\frac{S_{11}}{S_{21}} \sim C(z) = \sum_{k=0}^{N} c_k\, z^{-k} \qquad (17)$$

It is worthwhile to observe that the sequences of the coefficients $\{b_k\}$ and $\{c_k\}$ given in (5) and (17) respectively, can be interpreted as the discrete spectra of the periodic signals $T_{21}(\theta)$ and $C(\theta)$. As a consequence, one can write

$$c_k = h_k\, b_k \quad \text{with } k = 0 \sim N \qquad (18)$$

where the sequence $\{h_k\}$ defines the transfer function of the linear system of Fig.2. On the basis of this identification it is possible to characterize the linear system and then to define a new polynomial $T_{21}$ that must be synthesized, in order to obtain the desired frequency response $C^{(target)}(\theta)$:

$$b_k^{(new)} = c_k^{(target)}/h_k \quad \text{with } k = 0 \sim N \qquad (19)$$

# 3 Results

An example of application of the previously presented synthesis procedure is reported in this section. The case refers to the design of an equiripple 8-pole E-plane metal insert filter in a brass WR90 waveguide (insert thickness 0.52 mm). The bandwidth of 800 MHz is centered at 11. GHz, with a return loss of 25 dB. Starting from a Chebyshev polynomial, the synthesis procedure yielded the following data: $\gamma_k = \{50.7778, 19.9927, 13.8806, 12.7208, 12.4859\}$ deg, with $k = 0 \sim 4$; $\gamma_k = \gamma_{8-k}$ with $k = 5 \sim 9$ and $\psi_k = 0$ with $k = 1 \sim 8$. The full-wave analysis of the structure selected by the synthesis did not completely match the specifications because of the previously mentioned degradations effects. Hence, the linear system identification of the whole process allowed one to modify the polynomial $T_{21}$ that had to be synthesised and the following new data set was obtained: $\gamma_k = \{48.6337, 19.1474, 14.0873, 13.0088, 12.7972\}$ deg, with $k = 0 \sim 4$; $\gamma_k = \gamma_{8-k}$ with $k = 5 \sim 9$ and $\psi_k = \{1.6558, 2.2345, 2.4179, 2.4580\}$ deg, with $k = 1 \sim 4$; $\psi_k = \psi_{9-k}$ with $k = 5 \sim 8$. One should note that this

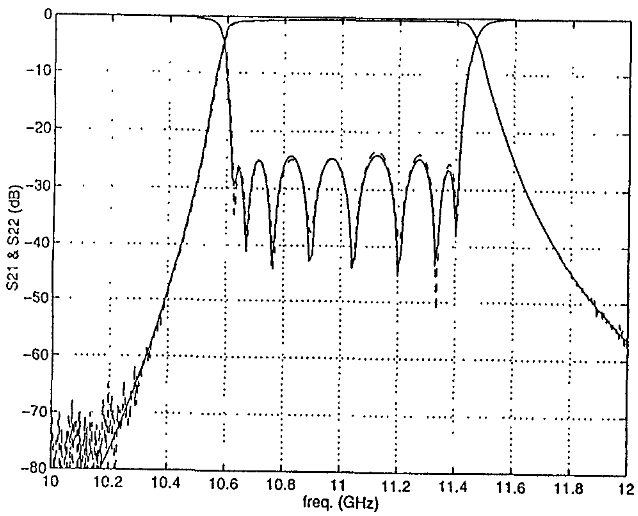


Figure 3: Eight-pole E-plane metal-insert WR90 waveguide filter. Measured transmission and reflection coefficients (dashed line). Full-wave analysis (solid line).

second synthesis step yielded non zero values of $\psi_k$ to compensate the different behaviour of the phase of the reflection coefficients $S_{11}^{(k)}$ of the junctions. The geometry of the whole filter was therefore selected as follows: septum lengths $s_1 = s_9 = 1.083$ mm, $s_2 = s_8 = 6.609$ mm, $s_3 = s_7 = 8.514$ mm, $s_4 = s_6 = 9.008$ mm, $s_5 = 9.110$ mm; resonator lengths $l_1 = l_8 = 11.515$ mm, $l_2 = l_7 = 11.529$ mm, $l_3 = l_4 = l_5 = l_6 = 11.527$ mm. Fig.3 shows the full-wave analysis of this configuration (dashed line) and the excellent agreement between the measured and predicted results. The analysis was carried out using the moment method where the resistivity of the brass was taken into account in the application of the boundary conditions. It should be noted that, even though the synthesis procedure does not deal with loss devices, the linear system identification process allows one to take them into account, thus yielding an excellent prediction of the insertion losses, as shown in Fig.3.

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